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Stable walking with asymmetric legs

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Abstract

Asymmetric leg function is often an undesired side-effect in artificial legged systems and may reflect functional deficits or variations in the mechanical construction. It can also be found in legged locomotion in humans and animals such as after an accident or in specific gait patterns. So far, it is not clear to what extent differences in the leg function of contralateral limbs can be tolerated during walking or running. Here, we address this issue using a bipedal spring-mass model for simulating walking with compliant legs. With the help of the model, we show that considerable differences between contralateral legs can be tolerated and may even provide advantages to the robustness of the system dynamics. A better understanding of the mechanisms and potential benefits of asymmetric leg operation may help to guide the development of artificial limbs or the design novel therapeutic concepts and rehabilitation strategies.

1. Introduction

For locomotion, humans usually select the most efficient bipedal gait, namely walking [1]. Besides efficiency, an equally important property of the chosen gait is stability. Human walking has inspired engineers to build bipedal robots, which should walk as stably as humans. Humans stabilize walking in a rather unconscious and intuitive way. In artificial legged systems, it is still a challenge to achieve stability during the highly dynamic gait.

The symmetry between contralateral legs is considered a useful condition for achieving stable locomotion. However, human legs are adaptable and offer many more possibilities than used during walking, such as when moving over a slightly uneven terrain or walking up- and downstairs. Additionally, human legs have to deal with unexpected situations, for example, if an unexpected step down arises. However, in most cases, leg mechanics and the neural system are able to master such critical situations.

The human locomotor system is not only able to deal with external disturbances, but it may also need to manage internal challenges. One such issue is the asymmetry between the left and right leg, which is found prominently in people with prostheses or orthoses. Here, it is obvious that the dynamics of both legs are different. One reason could be the differences in leg masses, which results in gaits of periods greater than 2, as predicted in a compass-gait model [2]. Interestingly, a left–right asymmetry during locomotion is also observed in able-bodied human subjects, even with equal leg masses [3]. Herzog et al have reported that asymmetries of ground reaction force (GRF) of about 4% are observable in normal walking [4]. Another common issue found in humans is leg length inequality [5]. The left–right differences in gait patterns are most visible for slower speeds [6, 7]. Besides left–right asymmetries, small stochastic stride-to-stride fluctuations are reported [8], which could also contribute to dynamical asymmetries of the gait. Stride-to-stride fluctuations might even be important to increase robustness of walking while minimizing the energetic cost [8]. In artificial walking devices, such fluctuations can be used for learning algorithms that improve robustness and top speed [9, 10].

These observations indicate that gait asymmetries are a key feature of human motion. Nevertheless, during the design process of a bipedal robot, engineers take care to make identical leg properties and joint control on both sides. But in practice, the dynamic properties of the legs are often not exactly equal, for instance the wear at the joints could be different. Such imperfections in the hardware (mechanics, actuators) could lead to a left–right asymmetry of leg dynamics. In this study, we investigate to what extent the asymmetric leg function may challenge walking stability.
Figure 1. The bipedal PogoWalker in (a) with compliant telescoping legs walking on the instrumented treadmill. The schematic in (b) illustrates the motor configuration for one leg.

Figure 2. Vertical ground reaction forces of PogoWalker measured with treadmill sensors. The forces are separated for both legs and were recorded with 200 Hz. Here, only the first ten strides are shown.

For this, we use a simulation model, i.e. the bipedal spring-mass model, that resembles the dynamics, and therefore the resulting center of mass (CoM) kinematics of human walking [11]. Following the concept of Full and Koditschek [12], we aim at subsequently increasing the complexity of a gait template such that the scientific question can be addressed while features of the underlying body dynamics are preserved. Here, the additional complexity is achieved by introducing asymmetry parameters into the model.

Human walking is characterized by single- and double-support phases, while the body is lifted during single-support around mid-distance. It is further distinguished from other gaits by the pattern of ground reaction force, where a double-humped pattern with two force maxima is observed [13]. Although human and artificial legs are very complex in their structure, their function during walking can be described mechanistically in a surprisingly simple way. At a preferred walking speed, a fairly linear force–length relationship is found [13]. Thus, the human leg can be understood as a simple prismatic leg spring supporting the body.

This paper is organized as follows. To motivate our study, we give two examples of asymmetric walking in the next section. In section 3, the methods used in our investigations are presented. In section 4, the results of simulations are described followed by their discussion in section 5.

2. Motivation

2.1. Robot experiment

One motivation for this study is the customized conceptual bipedal walking robot, PogoWalker (figure 1(a)). The legs of PogoWalker have the same stiffness and length. Each leg is moved by two motors: the first one actuates the hip and the second one shortens the leg during the swing phase (figure 1(b)). The motors for the leg rotation are arranged in the front and the back of the upper body to align the center of mass in the geometric center. As there was a slight mechanical coupling between leg rotation (first motor) and leg shortening (second motor), this construction approach led to different leg angles of the left and right leg. Due to the shifted leg placements, the leg compressions vary, which results in different maxima of the ground reaction forces (figure 2).
2.2. Prosthetic walking

In human walking, asymmetric gaits are typically found when the leg function is restricted due to amputation of a limb. Here, five subjects with unilateral transfemoral amputations were analyzed when walking at 1.1 m s\(^{-1}\) on an instrumented treadmill (ADAL-WR, HEF Tecmachine, Andrezieux, Boutheon, France) with integrated 3D force sensors (Kistler, Winterthur, Switzerland). For each subject, two trials were obtained, starting with the computerized artificial knee joint C-Leg, followed by the non-computerized 3R80 (both Otto Bock HealthCare). The asymmetry between both limbs was observed in the pattern of the vertical and horizontal GRFs (figure 3) with longer contacts on the intact side. All subjects were able to walk by selecting an asymmetric gait pattern.

2.3. Aims of this study

As demonstrated by the examples described above, asymmetric walking is a commonly observed gait pattern in both humans or artificial legged systems. So far, it is not clear to what extent asymmetry should be avoided or whether for certain conditions gait asymmetry could be accepted. For instance, Hof et al suggested that symmetric leg function is not necessarily required to be an aim of gait rehabilitation [14].

In order to approach this question, we will use a conceptual model for bipedal locomotion based on spring-like leg function [11] to investigate effects of asymmetric leg parameters on stability of walking patterns. In order to facilitate the comparison of the model with robot data, we used similar leg parameters in the model as in the PogoWalker. We expect that within certain ranges, asymmetric leg configurations are tolerated in the walking model, while stability is expected to decrease with increasing gait asymmetry.

3. Methods

Since walking with symmetric legs is a reference for our study, we first introduce the symmetric spring-mass model.

3.1. Symmetric model

The bipedal spring-mass model (figure 4) consists of two linear massless leg springs and a point mass \(m\) representing the center of mass (CoM) of the body. In the symmetric model, both leg springs are assumed to have the same properties: the rest length \(L_0\) and the stiffness \(k_0\) (figure 4). During the step, the system energy \(E\) remains constant. The location and velocity of the CoM are given by

\[
\begin{align*}
\mathbf{r} &= (x, y)^	op, \\
\mathbf{v} &= (\dot{x}, \dot{y})^	op,
\end{align*}
\]

The control consists of three phases. In the stance phase, the leg is retracted with a speed matching the treadmill speed. After take-off, the leg is actively shortened and protracted until \(\alpha_0 = 70^\circ\) is reached. In the last phase, the leg remains in this position until touch-down. The ground contact of the leg is detected by foot force sensors.

Since the gait pattern is not predefined, PogoWalker needs some steps to develop a continuous gait pattern. After some introducing steps, the robot adapts to the motion and shows asymmetrical vertical GRF patterns (figure 2). Even so, PogoWalker is able to walk over a considerable number of steps (>100) without stumbling.

The robot has two equal prismatic legs with fixed rest length of \(L_0 = 0.58\) m and fixed average leg stiffness of \(k_0 = 20\). The leg angle \(\alpha_0\) was adjusted at about 70\(^\circ\). The mass of PogoWalker is 4.1 kg, which is concentrated in the upper body. During the experiments, about 10% of the robot weight was suspended by elastic cords as a prevention from falling.

PogoWalker walked on an instrumented treadmill (Tecmachine, Andrezieux Boutheon, France) with integrated 3D force sensors (Kistler, Winterthur, Switzerland). In this study, the treadmill speed of 0.46 m s\(^{-1}\) was used. Thus, the average dimensionless energy in this experiment with PogoWalker was about 1.017.

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double-support phases, respectively. The points on the center of mass (CoM) trajectory show events vertical leg orientation (VLO), touch-down (TD) and take-off (TO). Black and gray parts of the CoM trajectory represent single- and double-support phases, respectively.

where \( r_{FP_i} \) is the position of the foot point \( FP_i \) of leg \( i \). In the swing phase of leg \( i \), the force \( F_i \) is zero. The transition from the swing phase to the stance phase (touch-down) occurs when the landing condition \( y = L_0 \sin(\alpha_0) \) is fulfilled, where \( \alpha_0 \) is the angle of attack (figure 4). The transition from the stance phase to the swing phase (take-off) of leg \( i \) occurs when the extending leg length reaches \( L_0 \).

Any gait of the spring-mass model is completely characterized by three fundamental system parameters (i.e. the dimensionless leg stiffness \( \tilde{k}_0 = (k_0L_0)/(mg) \), the angle of attack \( \alpha_0 \), the dimensionless system energy \( \tilde{E}_0 = E/(mgL_0) \)) and the four-dimensional vector \( \mathbf{r}_0 \) of initial conditions [11]. We start the simulations at the instant of the vertical leg orientation (VLO [15]) during single support. Here, the number of independent initial conditions can be reduced to 2, i.e. the height \( y_0 \) and the velocity angle

\[
\theta_0 = \arctan\left(\frac{y_0}{x_0}\right). \tag{3}
\]

The model parameters \( L_0 \) and \( m \) are chosen according to human data (\( L_0 = 1 \) m, \( m = 80 \) kg). The average dimensionless stiffness of PogoWalker legs is \( \tilde{k}_0 = 20 \). Using dimensional analysis [11], we convert \( k_0 \) to the dimensionless stiffness \( \tilde{k}_0 \) with respect to the model parameters \( m \) and \( L_0 \) and round it to 16 \( \text{kN m}^{-1} \).

The average dimensionless energy \( \tilde{E}_0 \) in the PogoWalker experiments is about 1.017. However, for this energy the bipedal symmetric model has no stable periodic solutions [15]. For this, we did all calculations in our investigation with constant dimensional energy \( E = 820 \) J corresponding to the dimensionless energy \( \tilde{E}_0 \) of 1.045. The difference between the system energy in experiment and one in simulations is less than 3\%, which is still within an acceptable range of tolerance.

The model is implemented in MATLAB (The MathWorks Inc., Natick, MA, USA). The differential equations are solved using the Runge–Kutta method (ode45) with absolute and relative tolerance of \( 10^{-13} \). Unless otherwise mentioned, the steps of \( \alpha_0 \) of 0.1° were used in all our calculations.

3.2. Asymmetric model

To investigate the asymmetric behavior, each leg is described by a different parameter set. For this, we introduce the
asymmetry parameters \( \varepsilon_a, \varepsilon_k, \varepsilon_L \) and \( \varepsilon_k \) of \( a_0, k_0, L_0 \) and \( \tilde{E}_0 \), respectively. \( \varepsilon_a, \varepsilon_k, \varepsilon_L \) and \( \varepsilon_k \) are also called imperfection or perturbation parameters [16]. For leg 1, the perturbations are subtracted from the corresponding control parameters (figure 5):

\[
\alpha_1 = \alpha_0 - \varepsilon_a, \quad k_1 = k_0 - \varepsilon_k, \quad L_1 = L_0 - \varepsilon_L, \quad \tilde{E}_1 = \tilde{E}_0 - \varepsilon_k.
\]

For leg 2, the perturbations are added (figure 5):

\[
\alpha_2 = \alpha_0 + \varepsilon_a, \quad k_2 = k_0 + \varepsilon_k, \quad L_2 = L_0 + \varepsilon_L, \quad \tilde{E}_2 = \tilde{E}_0 + \varepsilon_k.
\]

We call \( \varepsilon_a \) \( \alpha \)-asymmetry, \( \varepsilon_k \) \( k \)-asymmetry, \( \varepsilon_L \) \( L \)-asymmetry and \( \varepsilon_k \) consequently \( \tilde{E} \)-asymmetry.

The \( \tilde{E} \)-asymmetry \( \varepsilon_k \) affects the dimensional stiffness \( k_0 \) and the leg length \( L_0 \). However, the values of \( \varepsilon_k \) were chosen in such way that the dimensionless stiffness \( \tilde{k}_0 \) remains equal for both legs. For each \( \varepsilon_k \), the corresponding value of \( \varepsilon_L \) was calculated using \( \tilde{E}_0 = E/(mgL_0) \) with constant dimensional energy \( E = 820 \) J. Next, the stiffness asymmetry \( \varepsilon_k \) was calculated using \( \tilde{k}_0 = (k_0L_0)/(mg) \) with \( \tilde{k}_0 = 20 \). Note that although the dimensionless system energy \( \tilde{E} \) is different for the left and right leg, the dimensional energy \( E \) remains the same for both legs.

We consider positive perturbations of leg asymmetry only \( (\varepsilon_a > 0, \varepsilon_k > 0, \varepsilon_L > 0 \) and \( \varepsilon_k > 0 \)\). Switching the order of the steps, i.e. beginning with step 2 followed by step 1, we achieve the case of negative perturbation with exactly the same stability behavior.

In this study, we used steps of \( \varepsilon_a, \varepsilon_k, \varepsilon_L \) and \( \varepsilon_k \) as 0.1\(^\circ\), 0.1 kN m\(^{-1}\), 10\(^{-4}\) m and 10\(^{-4}\), respectively.

3.3. System analysis

Stability is one of the most important properties of bipedal walking. Here, we give a short description of the stability analysis used in our investigations.

The system analysis of a single step is described in detail in [15]. However, the investigation of walking in the asymmetric model requires the observation of a complete

Figure 6. Initial VLO conditions \( y_0 (a, b) \) and \( \theta_0 (c) \) of periodic walking patterns of the symmetric bipedal spring-mass model dependent on the angle of attack \( a_0 \). The thick lines indicate stable solutions. The diamonds \( \diamond \) at \( a_0 = 63.0^\circ, 63.4^\circ \) and 71.8\(^\circ\) represent bifurcation points \( p_1, p_2 \) and \( p_3 \), respectively, connecting branches of periodic solutions. Examples of CoM trajectories and vertical ground reaction forces corresponding to the points \( a_1, b_1, b_2, d_1 \) and \( d_2 \) are shown in figure 7.
Figure 7. Examples of CoM trajectories (upper row) and vertical ground reaction forces (vGRF) of both legs (lower row) corresponding to the points $a_1$, $b_1$, $b_2$, $d_1$ and $d_2$ in figure 6. The black points in the upper row indicate events vertical leg orientation (VLO), touch-down (TD) and take-off (TO) (figure 4).

Figure 8. Initial VLO conditions dependent on the reference angle of attack $\alpha_0$ for $\varepsilon_u = 2^\circ$, $4^\circ$ and $6^\circ$. The upper row shows the height $y_0$ of CoM at VLO. In the lower row, the velocity angle $\theta_0$ at VLO is represented. The gray curves are the initial conditions of the reference periodic gait patterns (see figure 6). The thick lines indicate stable solutions. The diamonds $\Diamond$ are bifurcation points connecting each of the two branches of periodic solutions.
gait cycle, comprising two subsequent steps. For this, slight modifications of the system analysis had to be done. Since each gait pattern with single-step periodicity is also a double-step periodic pattern, these modifications do not affect the analysis of single-step periodic gaits.

Step 1 starts in VLO1 defined by the initial conditions. It lasts until VLO2 is reached. The gait cycle is completed when VLO3 is reached (figure 4).

With VLO as the Poincaré section, we apply the Poincaré return map $F$. If $s_i = (y_i, \theta_i)$ is the state of the system in VLO, then after the complete gait cycle the state in VLO$_{i+2}$ is $s_{i+2} = F(s_i)$. Using the Poincaré map, we identify periodic walking solutions, which are represented as fixed points $s^*$ in the map $s^* = F(s^*)$. We calculate the fixed points as zeros of the function

$$G(s) = s - F(s)$$

applying a Gauss–Newton algorithm using the MATLAB function `fsolve` with relative and absolute tolerance of $10^{-8}$.

To determine the stability of a fixed point we calculate the Jacobi matrix $J_F(s^*)$ of $F$ in $s^*$. The gait pattern corresponding to $s^*$ is stable if the magnitude of both eigenvalues of $J_F(s^*)$ is less than 1 [17].

4. Results

4.1. Period-1 gaits with symmetric legs

Periodic solutions of the symmetric model are shown in figure 6. We consider two kinds of single-step periodic walking patterns, branches A and B, which were already presented in [15]. The vertical ground reaction forces and CoM trajectories of patterns on A are mirrored at VLO (example $a_1$ in figure 7 corresponding to the point $a_1$ in figure 6), in contrast to solutions of branch B ($b_1$ and $b_2$ in figures 6 and 7).

The branches A and B are connected by the transcritical bifurcation point $p_3$ at $\alpha_0 = 71.8^\circ$. On branch A, the stable...
solutions are found between $\alpha_{\text{min}} = 69.4^\circ$ and the bifurcation point $p_\text{a}$. The stable solutions are continued by branch B starting at the bifurcation point $p_\text{a}$ and ending at $\alpha_{\text{max}} = 74.7^\circ$. For parameter values $\alpha_{\text{min}}$ and $\alpha_{\text{max}}$, the eigenvalues of the Jacobi matrix are complex conjugate with magnitude equal to 1. Together with the change of stability, this indicates that in $\alpha_{\text{min}}$ and $\alpha_{\text{max}}$, Hopf bifurcations occur [18].

In the following, we investigate the size of the region where locomotion is stable. Therefore, we define the continuous range of stable solutions $\Delta \alpha = \alpha_{\text{max}} - \alpha_{\text{min}}$ and call it the $\alpha$-range. The symmetric model exhibits an $\alpha$-range of $\Delta \alpha = 5.3^\circ$, which provides a reference for investigations on the asymmetric model (sections 4.3, 4.4 and 4.5).

4.2. Period doubling

Additionally, we present two kinds of double-step periodic patterns lying on branches D1 and D2 in figure 6(b). D1 and D2 are connected to branch A by the bifurcation points $p_\text{a}$ and $p_\text{b}$. Periodic solutions lying on branches D1 and D2 were calculated using steps of $\alpha_0$ of 0.01°.

Starting at VLO, after the first step a fixed point lying on D1 reaches in VLO$_{\alpha_0}$ the corresponding point on the contrary part of D1 (example $d_1$ in figures 6 and 7). For example, the trajectory of $d_1 = (0.978 \text{ m}, 11.03^\circ)$ has in VLO$_{\alpha_0}$ a different CoM height $y_{\alpha_0} = 0.987 \text{ m}$ and velocity angle $\theta_{\alpha_0} = -11.943^\circ$ (figure 6(c)). After the second step, the start point on D1 is reached and the periodicity is fulfilled.

The patterns on D2 are characterized by $(y_{\alpha_0}, \theta_{\alpha_0}) = (y_{\alpha_0}, -\theta_{\alpha_0})$, i.e. after one step these gaits have the same VLO height $y_{\alpha_0}$ and exactly the opposite sign of the velocity angle $\theta_{\alpha_0}$ (figure 6(c)).

All double-step periodic patterns on D1 and D2 are unstable. Moreover, the magnitude of the velocity angle $\theta_{\alpha_0}$ increases rapidly (figure 6(c)), which makes this pattern very sensitive to all kinds of asymmetric perturbations. In most cases, leg asymmetry causes the take-off of the supporting leg in a single-support phase. Since our definition of walking requires at least one leg always having ground contact, this locomotion pattern can no longer be treated as walking. For this, further discussion of the influence of leg asymmetry on the walking patterns lying on D1 and D2 is omitted.

4.3. Asymmetry of $\alpha_0$

As can be seen in figure 8, moderate perturbations of $\alpha_0$ do not affect the bifurcation. In this case, $\epsilon_\alpha$ is a bifurcation preserving imperfection [19]. As long as this bifurcation exists, stable solutions can be found on both branches, A and B.

The branch B diminishes with increasing asymmetry. For $\epsilon_\alpha > 5^\circ$, no bifurcation and no second branch could be found. For perturbations larger than 9.9°, no periodic solutions at all were determined. Compared to the symmetric case, we observed an increase of the $\alpha$-range for values of $\epsilon_\alpha$ less than 4.5°. The maximum value of $\Delta \alpha = 6.9^\circ$ was found at $\epsilon_\alpha = 2.8^\circ$ (figure 9(a)). Here, the left limit of the region of stable solutions is $\alpha_{\text{min}} = 68.8^\circ$ and the right one is $\alpha_{\text{max}} = 75.7^\circ$ (figure 9(b)). For $\epsilon_\alpha > 3^\circ$, the $\alpha$-range monotonically decreases due to the reduction of the branch B.

For a small range of $\alpha_0$ around $70.3^\circ$ and an $\alpha$-asymmetry of 7.8°, the walking gait not only remains stable (figure 9(b)), but the stability, i.e. reflected by the magnitude of both eigenvalues, is also improved (figure 10). For this value of $\alpha_0$, stable walking is still possible with the total left–right deviation of angle of attack of $15.6^\circ$.

One interesting result is that $\alpha$-asymmetry can stabilize previously unstable symmetric walking patterns. For example, the unstable symmetric gait for $\alpha_0 = 69^\circ$ becomes stable for values of $\epsilon_\alpha$ greater than $2.2^\circ$ and less than $5^\circ$ (figure 9(b)).

The main reason for vanishing of periodic solutions for higher values of $\alpha_0$ is that the length of the supporting leg spring in a single-support phase reaches the rest length $L_0$. Like in the case of double-step periodic patterns, we observe a rapid increase of magnitude of the velocity angle $\theta_{\alpha_0}$ (figure 8) and an appearance of a flight phase during the step.

4.4. Asymmetry of $k_0$

Adding leg stiffness asymmetry $\epsilon_k$ to the system results in two branches of periodic solutions without any bifurcations.
4.5. Asymmetry of $L_0$ and $\tilde{E}_0$

The rest length asymmetry $\varepsilon_L$ affects the system in the similar way as leg stiffness asymmetry $\varepsilon_k$ (section 4.4). All stable solutions of the asymmetric system lie also on one branch between two Hopf bifurcations (figure 13). We observe no increase in the $\alpha$-range (figure 14(a)). The stable solutions are lost at $\varepsilon_L = 9$ mm and periodic solutions are lost at $\varepsilon_L = 21$ mm (figure 14(b)). Hence, considering the reference leg length $L_0$ of 1 m, stable walking exists for the total difference in the length of contralateral legs of up to 1.8%.

A similar tolerance of 2% is predicted for asymmetries in dimensionless energy $\varepsilon_{\tilde{E}}$. Here, stable solutions exist for the values of $\varepsilon_{\tilde{E}}$ up to 0.01. Periodic solutions are lost at $\varepsilon_{\tilde{E}} = 0.023$.

5. Discussion

In this study, we investigated the effect of asymmetries in the angle of attack $\varepsilon_\alpha$, leg stiffness $\varepsilon_k$, leg length $\varepsilon_L$ and dimensionless energy $\varepsilon_{\tilde{E}}$ between both legs on the dynamics and stability of spring-mass walking. We have demonstrated that the asymmetric leg function does not necessarily reduce the region of stable walking. The $\alpha$-asymmetry can not only be tolerated during walking but it may also result in advantages, as demonstrated by the increased $\alpha$-range (figure 9). For a small range of values of $\varepsilon_\alpha$, the $\alpha$-asymmetry can even stabilize symmetric walking gaits. Surprisingly, for values of $\alpha_0$ around 70°, after applying $\alpha$-asymmetry to the symmetric system the gait not only remains stable, but stability may also be improved (figure 10). This indicates that asymmetric gaits could be a better solution for asymmetric leg configuration as was already suggested for amputees [14].

There are specific effects of asymmetries in $\alpha_0$, $k_0$, $L_0$ and $\tilde{E}_0$ on the region of stable walking. With increasing asymmetry of leg stiffness $\varepsilon_k$, of leg length $\varepsilon_L$ and of dimensionless energy $\varepsilon_{\tilde{E}}$, the $\alpha$-range $\Delta \alpha$ diminishes continuously. Moreover, with $\varepsilon_\alpha$ stable solutions were found as long as periodic solutions existed, while with $\varepsilon_k$, $\varepsilon_L$ and $\varepsilon_{\tilde{E}}$ stability was lost even though periodic solutions were still present.
Experimental data on human walking [20–22] show that vertical GRFs of the longer leg are larger than the GRFs of the shorter leg. This is in agreement with our model predictions (figure 15). However, the analyzed walking model is very sensitive to the \( L \)-asymmetry. In the spring-mass model, the leg length \( L_0 \) has not only influenced the leg dynamics during the stance but also affected the instances of touch-down and take-off.

The effects of \( \tilde{E} \)-asymmetry are similar. This could be due to the fact that \( \tilde{E}_0 \) is directly affected by \( L_0 \). Additionally, \( \tilde{E}_0 \) is also affected by \( k_0 \). Hence, the effects of \( \tilde{E} \)-asymmetry could further depend on the selected leg stiffness \( k_0 \) or system energy \( E \). This needs to be investigated in more detail.

In humans, even small deviations of the leg length can cause stress fractures, back pain and osteoarthritis [23]. However, because of the flexibility of human legs, small leg length discrepancies are not expected to affect the stability of walking in such a crucial way. We suppose the missing leg segmentation in the spring-mass model as one of the reasons for the high sensitivity with respect to \( L \)-asymmetry. Extending the spring-mass model by a knee joint [24] or by a foot segment [25] could increase the range of stable solutions. Other possible ways to improve the stability behavior of the system under influence of \( \varepsilon_L \) could be swing leg control as described by [26–28] or suitable combinations of \( L \)-asymmetry with asymmetry of \( \alpha_0 \) or asymmetry of \( k_0 \).

Humans with asymmetric leg mechanics often walk shifting their body weight from one leg to another in the lateral plane. For instance, the medial−lateral acceleration of the CoM at touch-down is greater for the short leg, indicating a faster transfer of the mass to the shorter extremity [29]. For this, an extension of the spring-mass model into the lateral plane could predict appropriate methods to manage the disadvantages of asymmetric walking. Since humans avoid narrow step widths, because they are less stable [30], the suitable lateral foot placement could additionally stabilize asymmetric gaits. First three-dimensional symmetrical models for running [31, 32] and walking [33] already exist.

Except for the \( \alpha \)-asymmetry \( \varepsilon_\alpha \), all stable solutions of the asymmetric system for one set of parameters lie between two Hopf bifurcation points (figures 11 and 13). In the case of \( \varepsilon_\alpha \), a part of the region of stable solutions is cut off (figure 8). Here, larger left−right deviations of \( \varepsilon_\alpha \) lead to the flight phases caused by take-offs during single support. The resulting gaits are skipping, i.e. gaits which contain double-support phases along with flight phases [34, 35]. Hence, allowing short air phases in walking patterns of a robot with asymmetric leg configuration could enlarge its range of stable gaits.

Despite the tolerance and increasing stability of the system toward \( \alpha \)-asymmetry, there is one important disadvantage of asymmetric gaits. As is shown in [36], asymmetric walking is more energy consuming than symmetric walking. However, it is still not clear how asymmetries of \( \alpha_0 \), \( k_0 \), \( L_0 \) and \( \tilde{E}_0 \) affect the energetic costs of periodic walking patterns. Although the model is conservative, the mechanical work of the legs during contact can be calculated and used as an estimation of energetic efficiency [15]. This reflects the situation that legs are not just passive springs but need to be actuated with muscles or motors. Hence, only part of the work predicted by the spring-mass model can be done completely passively by springs. The actual percentage of passive versus active work during walking will depend on the way actuators and springs are arranged and operated during locomotion. This would require an extension the model, which was not envisioned in this study.

Additionally, by changing either the leg stiffness [37] or leg length [38] during ground contact, an additional energy input can be simulated. Such an actuated asymmetric model could be an important tool for investigations of asymmetric energy supply in the contralateral legs. For example, an actuation of the ankle joint with peak positive work prior to take-off could significantly improve performance of active prostheses or orthoses [39–41].

In our study, the model is assumed to be deterministic, i.e. all parameters and states are precisely defined. However, this contradicts the nature of human gait where leg parameters and system states do change over time. In order to estimate the ‘global’ stability, as indicated by a risk of falling, it would be required to take the uncertainties of the system into account. This could be done based on stochastic models, e.g. by introducing metastability as suggested by [42].
The walking model used in our study is currently the best candidate for a template of human walking [44, 45, 46]. It is able to predict walking patterns which are periodic even with asymmetric leg configurations. Hence, humans could benefit from such solutions by appropriately adjusting leg parameters. However, the model is much too simple to indicate how these patterns can be achieved in a neuro-muscular system.

Even though the model predicts stable walking solutions based on asymmetric compliant leg configurations, it needs to be proven in future studies whether such mechanically attractive behavior indicated by the template model has any practical advantage. In order to address these issues, experimental studies on asymmetric gaits are required and need to be compared to the biomechanical model of increased complexity, taking neuronal and muscular mechanisms into account.

Gait asymmetries naturally occur in both human walking and technical walking systems (e.g. legged robots, prosthetic legs). The predictions of the model may help to estimate the tolerated range of asymmetries depending on the overall leg properties (e.g. leg stiffness, leg length) and the gait characteristics (e.g. walking speed, angle of attack). This could help to derive procedures to indicate when differences in the leg function may threaten the overall gait stability.

As the leg angle is a leg parameter which is adjustable in the PogoWalker, we focused on analyzing the effects of different leg angles on walking stability with asymmetric legs. In future, similar considerations should also include changes in leg stiffness and leg length in order to calculate the corresponding $k$-range and $L$-range, representing the range of parameters resulting in stable walking. Future studies need to show how asymmetries affect stable $k$-range and $L$-range. Moreover, suitable combinations of leg asymmetries (e.g. $\varepsilon_\alpha$, $\varepsilon_k$, $\varepsilon_L$ and $\varepsilon_E$) could allow additional effects influencing the range of stable walking. Also the proper selection of the control parameters ($\alpha_0$, $k_0$, $L_0$, $\tilde{E}_0$) could further enhance
the tolerated range of asymmetries $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_L$, and $\varepsilon_R$. These effects need to be studied in more detail.

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